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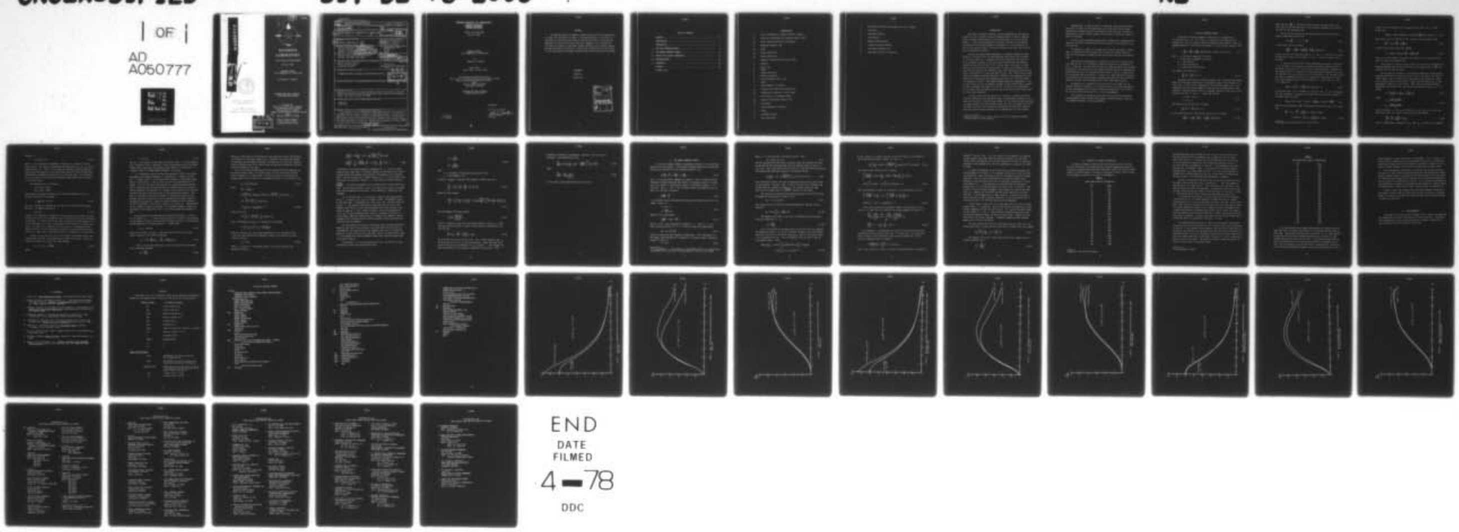

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MOMENTUM THEORY OF A PROPELLER IN A SHEAR FLOW

by Theodore R. Goodman

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MOMENTUM THEORY OF A PROPELLER IN A SHEAR FLOW

by

Theodore R. Goodman

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ABSTRACT

The momentum theory of moderately loaded propellers has been generalized to account for the effects of shear in the oncoming stream. A condition to determine the distribution of thrust for optimum efficiency is derived which generalizes a condition derived by Betz for a propeller in a uniform flow. This condition leads to a nonlinear integral equation, and a computer program has been developed for solving it. Results for a propeller of a given radius in particular wakes at different Reynolds numbers are presented and discussed.

KEYWORDS

Propeller

Shear Flow

TABLE OF CONTENTS

NOMENCLATURE

a	axial interference or induction factor = $u_\infty/2U_\infty$
a'	rotational interference or induction factor = $\omega/2\Omega$
C_T	thrust loading coefficient = $T/\frac{1}{2}\rho\, U_\infty^2\, S$
d	propeller diameter = 2R
J_A	U_∞/nd
K_Q	torque coefficient
K_T	thrust coefficient
n	number of revolutions per second = $\Omega/2\pi$
p	pressure
P	power = ΩQ
Q	torque
r	radial coordinate
R	radius of propeller
S	area of propeller disc = πR^2
T	thrust
u	axial component of velocity
u_o	induced axial velocity at actuator disc
u_∞	induced axial velocity in the far wake
U	axial velocity in oncoming stream
U_R	velocity of oncoming stream at r=R
U_∞	ship speed
v	radial component of velocity
V	$2U-U_R$
w	azimuthal velocity
x	axial coordinate

δ	Dirac delta function; also symbol for first variation
η	efficiency
λ	Lagrange multiplier
ρ	fluid density
ψ	Stokes stream function
ω	induced rotational velocity
Δ	jump across actuator disc
Ω	rotational velocity of propeller

INTRODUCTION

The theory of propeller design is a well-developed art that has progressed for over a century from axial momentum theory to general momentum theory to blade element theory to lifting-line theory to lifting-surface theory. It has thus become possible to predict the mean radial pressure distribution with various degrees of refinement for a given propeller as well as the harmonic content, and also to determine the mean load distribution for optimum efficiency.

Despite the sophisticated mathematical treatments presented by many authors, all theories thus far developed have been based on the existence of a velocity potential. Propellers which are placed near the stern of a ship or submarine operate in the wake of the vehicle where the oncoming stream contains vorticity and this is usually taken into account by parametrically varying the wake velocity over the plane of the propeller and retaining the assumption of potential flow. This procedure can be justified only if the vorticity is weak (see Sears,[1*] p.42). Thus, the lifting surface theories presented by Morgan and Wrench,[2] Yamazaki,[3] and Greenberg,[4] are all based on this assumption, and Greenberg goes so far as to say that unless the assumption is made "..the problem becomes hopelessly unmanageable." This is very likely to be so for general unsteady lifting surface theory. However, it leaves unanswered the question as to the suitability of the assumption for an actual ship wake whose vorticity may or may not be sufficiently small to justify the assumption.

In those cases where the vorticity is not weak, a velocity potential cannot be assumed and the presence of shear in the oncoming stream will affect the flow field. Theories which account for the shear have been considered by several authors for the flow about wings, the first such paper being that of Karman and Tsien,[5] but the parallel development for propellers seems to have been ignored probably because of the unmanageability of the problem.

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*Superior numbers in text matter refer to similarly numbered references listed at the end of this report.

Nevertheless, it would be useful to have some simple solutions which do account for shear so that comparisons can be made with the weak shear approximation thereby allowing an assessment of the weak shear assumption. Two possible "simple" theories appear to be possible in which the effect of shear is included.

The first such is momentum theory which is basically a one-dimensional theory and for which the wake profile can be allowed to vary radially in an arbitrary way.

The second such theory is a steady state lifting-line theory in which shear is accounted for but the wake profile takes on a special shape for which the field equation becomes no more complicated than that for potential flow.

The analysis presented below is confined to the first of these simplified theories, but it is intended to develop the second in a subsequent report. In Section I the equations for the axial momentum theory are developed. In Section II the equations for the general momentum theory are developed and the condition of Betz for the optimum distribution of thrust is generalized to an arbitrary wake distribution. In Section III some numerical evaluations are presented for a measured wake and comparisons made with results obtained simply by varying the wake parametrically (weak shear approximation).

The analysis will generally follow the article by Glauert[6] for the momentum theory of a propeller in a uniform stream. This article synthesizes the main developments of the theory which have changed very little since it was written (but see Reference 7 for a recent development).

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I. THE AXIAL MOMENTUM THEORY

We consider the Euler equations of motion for an incompressible frictionless fluid. At $x=0$, there is a propeller to be modelled by an actuator disc that can sustain a pressure jump $\Delta p = p(x=0+) - p(x=0-)$. The axial momentum equation in cylindrical coordinates (x,r) for axially symmetric flow is

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{1}{\rho} \Delta p(r) \delta(x) \quad ; \Delta p(r) = 0 \text{ for } r > R . \qquad (1.1)$$

Here u is the axial component of velocity

v is the radial component of velocity

p is the pressure

ρ is the fluid density

δ is the Dirac delta function

The equation of continuity is

$$\frac{\partial}{\partial x} (ru) + \frac{\partial}{\partial r} (rv) = 0 \quad . \qquad (1.2)$$

Now assume there exists an oncoming stream $U(r)$, the so-called nominal wake, which has been generated by a body somewhat ahead of the actuator disc. Since $U(r)$ is taken to be independent of the azimuthal angle, we will be considering only the zeroth harmonic of the wake and the mean forces on the propeller. The perturbation velocity components imparted to the fluid by the actuator disc are u' and v' . Then, upon setting

$$\begin{aligned} u &= U(r) + u'(x,r) \\ v &= v'(x,r) \end{aligned} \qquad (1.3)$$

and substituting into Eq.(1.2), we obtain

$$\frac{\partial}{\partial x} (ru') + \frac{\partial}{\partial r} (rv') = 0 \quad . \qquad (1.4)$$

On the other hand, upon substituting into Eq.(1.1), we obtain

$$U \frac{\partial u'}{\partial x} + u' \frac{\partial u'}{\partial x} + \frac{dU}{dr} v' + \frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{1}{\rho} \Delta p(r) \delta(x) \qquad (1.5)$$

where the term $\frac{\partial u'}{\partial r} v'$ has been ignored as being of higher order. The third term on the left-hand side of this equation represents the effect of the shear of the oncoming stream.

The continuity equation can be satisfied identically by defining a Stokes stream function such that

$$rv' = \frac{\partial \psi}{\partial x} \quad ; \quad ru' = -\frac{\partial \psi}{\partial r} \tag{1.6}$$

in which case, Eq.(1.5) becomes

$$U \frac{\partial u'}{\partial x} + u' \frac{\partial u'}{\partial x} + \frac{1}{r}\frac{dU}{dr}\frac{\partial \psi}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{1}{\rho}\Delta p(r)\delta(x) \quad . \tag{1.7}$$

At this point we will drop the primes on u' and v' ; since we will rarely be required to refer to the original u and v , this will cause little confusion.

In order to solve for the velocity components everywhere in the field, it is, of course, also necessary to consider the radial momentum equation and solve it simultaneously with the axial momentum equation. However, for our purposes we will be content to consider the overall momentum of the system and with this view, only Eq.(1.7) will be required. Far upstream ($x=-\infty$), all velocity components vanish and the pressure will be taken to be zero. Thus, upon integrating Eq.(1.7) with respect to x from $x=-\infty$ to $x=0-$, it is found that

$$Uu(0-) + \frac{u^2}{2}(0-) + \frac{1}{r}\frac{dU}{dr}\psi(0-) + \frac{1}{\rho}p(0-) = 0 \quad . \tag{1.8}$$

On the other hand, if we integrate Eq.(1.7) from $x=0+$ to $x=\infty$, and assume that any difference in pressure in the wake from its value far upstream is of higher order,* we obtain

$$U[u_\infty - u(0+)] + \tfrac{1}{2}[u_\infty^2 - u^2(0+)] + \frac{1}{r}\frac{dU}{dr}[\psi_\infty - \psi(0+)] = \frac{p(0+)}{\rho} \tag{1.9}$$

Upon forming the quantity $\frac{\Delta p}{\rho}$ by adding Eq.(1.8) and Eq.(1.9), it is found that

$$\frac{\Delta p}{\rho} = U u_\infty + \frac{u_\infty^2}{2} + \frac{1}{r}\frac{dU}{dr}\psi_\infty + U[u(0-) - u(0+)]$$

$$+ \tfrac{1}{2}[u^2(0-) - u^2(0+)] + \frac{1}{r}\frac{dU}{dr}[\psi(0-) - \psi(0+)] \quad . \tag{1.10}$$

*indicated by von Mises [8] to be of third order.

Finally, if we integrate Eq.(1.7) across the disc from $x=0-$ to $x=0+$, we see that

$$U[u(0-) - u(0+)] + \tfrac{1}{2}[u^2(0-) - u^2(0+)] + \frac{1}{r}\frac{dU}{dr}[\psi(0-) - \psi(0+)] = 0 \tag{1.11}$$

Subtracting Eq.(1.11) from Eq.(1.10), we obtain for the pressure jump

$$\frac{\Delta p}{\rho} = U u_\infty + \frac{u_\infty^2}{2} + \frac{1}{r}\frac{dU}{dr}\psi_\infty \tag{1.12}$$

from which the thrust of the disc becomes

$$T = \int \Delta p dS = \rho \int \left[U u_\infty + \frac{u_\infty^2}{2} + \frac{1}{r}\frac{dU}{dr}\psi_\infty \right] dS \tag{1.13}$$

where dS is the element of area and the integration is to be carried out over the area of the actuator disc.

This formula is identical to the result for the thrust as obtained from ordinary momentum theory with the additional term proportional to $\frac{dU}{dr}$ that accounts for the effect of shear.

At this point we may obtain a relationship between the stream function and axial perturbation velocity at the disc and in the wake to the order of linear theory. Now in linear theory, the pressure must descend to $-\frac{1}{2}\Delta p$ just in front of the disc and rise to $+\frac{1}{2}\Delta p$ just behind the disc, so that the quantity $p(0+)+(p-)$ must vanish. Thus, upon taking the difference between Eq.(1.8) and Eq.(1.9), and ignoring the quadratic terms in u^2, we obtain

$$u_o + \frac{1}{rU}\frac{dU}{dr}\psi_o = \frac{1}{2}\left[u_\infty + \frac{1}{rU}\frac{dU}{dr}\psi_\infty\right] \tag{1.14}$$

where

$$u_o = \frac{u(0+) + u(0-)}{2}$$
$$\psi_o = \frac{\psi(0+) + \psi(0-)}{2} \tag{1.15}$$

Upon eliminating u_o and u_∞ in favor of ψ_o and ψ_∞ by virtue of the definition (1.6), it is easy to show that Eq.(1.14) becomes

$$\frac{d\psi_o}{dU} - \frac{\psi_o}{U} = \frac{1}{2}\left(\frac{d\psi_\infty}{dU} - \frac{1}{U}\psi_\infty\right) \tag{1.16}$$

This is a differential equation for ψ_o when ψ_∞ is given, and its general

solution is

$$\psi_o = \frac{1}{2}\psi_\infty + KU \tag{1.17}$$

where K is a constant of integration. However, for a propeller with no boss, the axis $r=0$ must be a streamline and hence both ψ_o and ψ_∞ vanish there; whence $K=0$. Thus to first order the perturbation stream function in the plane of the propeller is half the stream function in the ultimate wake, and hence the perturbation velocities are in the same ratio. This relationship is, of course, the same as had previously been deduced for a propeller in a uniform stream.

Now reconsider Eq.(1.11) and let

$$\Delta u = u(0-) - u(0+)$$
$$\Delta\psi = \psi(0-) - \psi(0+) \quad .$$

Upon ignoring the quadratic terms and utilizing the definition (1.6), it is easy to show that this becomes

$$-U\frac{d}{dU}(\Delta\psi) + \Delta\psi = 0 \quad . \tag{1.18}$$

This is a differential equation for the jump in the stream function across the disc. Its general solution is

$$\Delta\psi = K_1 U \quad . \tag{1.19}$$

where K_1 is a constant of integration. For a propeller with no boss, $\Delta\psi$ must vanish at $r=0$, while for a propeller with a boss, $\Delta\psi$ must vanish at the radius of the boss. In either case, K_1 must vanish, and hence we have proven that to first order there is no jump in the stream function across the actuator disc, that is, no mass flow is created at the disc, and as a consequence of the definition (1.6), the axial perturbation velocity is also continuous across the disc, a result that is the same as that for a propeller in a uniform flow.

Finally, we may cast Eq.(1.13) for the thrust into another form by considering that $dS=2\pi r dr$ and integrating the shear term by parts. Then, by virtue of the definition (1.6), the thrust becomes

$$T_a = \rho\int\left(Vu_\infty + \frac{u_\infty^2}{2}\right)dS \quad , \tag{1.20}$$

where

$$V = 2U - U_R \tag{1.21}$$

and U_R is the velocity of the wake at the outer radius R of the propeller disc. Equation (1.20) is identical in form to the expression for the thrust without shear except that the oncoming stream velocity is replaced by V, a quantity that will become negative whenever $U(r) < U_R/2$.

It is necessary to point out that the expression (1.20) is the thrust developed on the propeller because it is operating in a wake of velocity $U(r)$. In other words, it is the thrust that develops on a propeller tested behind a screen wake in a water tunnel test facility (neglecting wall effects). It is not the thrust that develops on a propeller mounted behind a ship or submarine with this wake because the induced velocity and pressure fields created by the propeller will increase the resistance on the body over and above that which would be present if the propeller were absent. Thus, when it is mounted on the ship or submarine, the propeller must not only overcome the body resistance but also the additional resistance induced on the body by the propeller. Hence, Eq.(1.20) represents only the apparent thrust of the propeller and for this reason it is called T_a. Determination of the induced resistance on the body is an important element in analyzing the propeller ship combination, but it is not the problem that concerns us here.

We now consider the power required to produce the thrust T_a which will be called P_a. It is clear that the power at any annulus will be equal to the element of thrust times the velocity $(U+u_o)$ at which it is being transported summed over all annular elements. In other words

$$P_a = \int (U+u_o)\, dT \quad . \tag{1.22}$$

Since to first order $u_o = \frac{1}{2} u_\infty$, then upon substituting for the thrust density from Eq.(1.12), there is obtained

$$P_a = \rho \int \left(U + \frac{u_\infty}{2}\right) \left[U u_\infty + \frac{u_\infty^2}{2} + \frac{1}{r}\frac{dU}{dr}\psi_\infty\right] dS \quad . \tag{1.23}$$

Finally, the apparent efficiency is given by the ratio of the useful work to the power absorbed:

$$\eta_a = \frac{U_\infty T_a}{P_a} \tag{1.24}$$

where U_∞ is the velocity of the propeller body combination (which unfortunately is not known for the present model since it depends on the connection between the body wake and the speed of advance). The apparent efficiency can, paradoxically, be greater than unity since we have not debited the propeller with the induced resistance on the body.

We will now determine the condition to be satisfied for the propeller of maximum efficiency. Thus, for a given thrust, we wish to minimize the power used to generate it. In other words, we wish to set the first variation of $P_a+\lambda U_\infty T_a$ equal to zero, where λ is a Lagrange multiplier. In Eq.(1.23), we set

$$\psi_\infty = -\int_0^r r_1 u_\infty(r_1)\,dr_1 \quad . \tag{1.25}$$

Then,

$$\delta P_a + \lambda U_\infty \delta T_a =$$

$$2\pi\rho \int_0^R \Big\{ U^2\delta u_\infty + 2Uu_\infty\delta u_\infty + \frac{3}{4}u_\infty^2\,\delta u_\infty - \frac{\delta u_\infty}{2r}\frac{dU}{dr}\int_0^r r_1 u_\infty(r_1)\,dr_1$$

$$-\left(U + \frac{u_\infty}{2}\right)\frac{1}{r}\frac{dU}{dr}\int_0^r r_1\,\delta u_\infty(r_1)\,dr_1$$

$$+ \lambda U_\infty(V\delta u_\infty + u_\infty\delta u_\infty)\Big\} r\,dr = 0 \quad . \tag{1.26}$$

Consider the term

$$-2\pi\rho \int_0^R \left(U + \frac{u_\infty}{2}\right)\frac{dU}{dr}\,dr \int_0^r r_1\,\delta u_\infty(r_1)\,dr_1$$

Upon interchanging the order of integration, this becomes

$$-2\pi \int_0^R r\delta u_\infty dr \int_r^R \left(U(r_1) + \frac{u_\infty}{2}\right)\frac{dU}{dr_1}\,dr_1 \quad .$$

Thus the variation δu_∞ may now be factored out of all the terms in Eq. (1.26), and upon applying the fundamental lemma of the calculus of variations and letting

$$u_\infty = 2aU_\infty \tag{1.27}$$

where a is the axial interference factor, we find the condition for optimum efficiency:

$$\frac{3}{2}\left(\frac{U}{U_\infty}\right)^2 + 4a\left(\frac{U}{U_\infty}\right) + 3a^2 - \frac{1}{r/R}\,\frac{dU/U_\infty}{dr/R}\int_0^{r/R}\frac{r_1}{R}\,a\,d\frac{r_1}{R}$$

$$-\frac{1}{2}\left(\frac{U_R}{U_\infty}\right)^2 - \int_{r/R}^{1} a\,\frac{dU/U_\infty}{dr_1/R}\,d\frac{r_1}{R} + \lambda\left[2\frac{U}{U_\infty} - \frac{U_R}{U_\infty} + 2a\right] = 0 \quad . \qquad (1.28)$$

In contrast to the case of parametric dependence on U, the axial interference factor, when shear is included, depends on conditions at all radial stations across the actuator disc because of the presence of the integral terms. One way to solve this integral equation for a is by iteration. The first step is to specify a value of λ. Then ignore the terms involving $\frac{d(U/U_\infty)}{d(r/R)}$. The resulting quadratic can be solved exactly at each radial station. The resulting values of $a(r/R)$ can then be used in the two integral terms and the quadratic solved again. This procedure is repeated to convergence.

It will be noted that it is necessary to specify the Lagrange multiplier λ, a quantity that is not, of course, known. The apparent thrust, on the other hand, is presumed given. Thus, once $a(r)$ has been found for a given λ, the apparent thrust may be calculated from Eq.(1.20), the apparent power from Eq.(1.23), and the apparent efficiency from Eq.(1.24). Hence, by varying λ over a range of values, a plot can be prepared of T_a vs. η_a for optimum apparent efficiency of a propeller of radius R in a given wake. Note that these curves will change if the propeller radius changes so that there may exist an optimum propeller radius especially for wakes that are not monotonic.

The distribution of the axial interference factor as obtained by this method is not likely to be realistic since rotational velocities are not accounted for, but the curves of ideal apparent efficiency vs. thrust and power will be very close to the curves when rotation is accounted for since it is well-known from ordinary momentum theory that rotation affects these curves only slightly.

The equations for thrust and power may be put into coefficient form by defining a thrust and torque coefficient:

$$K_T = \frac{T_a}{\rho n^2 d^4}$$
$$K_Q = \frac{P_a}{\Omega \rho n^2 d^5} \tag{1.29}$$

Here

n is the number of revolutions per second = $\Omega/2\pi$

d is the diameter = 2R

We define $J_A = U_\infty/nd$, then Eq.(1.20) becomes in coefficient form

$$\frac{K_T}{J_A^2} = \pi \int_0^1 \left[\left(2\frac{U}{U_\infty} - \frac{U_R}{U_\infty}\right) + a\right] a \frac{r}{R}\, d\frac{r}{R} \quad . \tag{1.30}$$

Equation (1.23) becomes

$$\frac{K_Q}{J_A^3} = \frac{1}{2}\int_0^1 \left(\frac{U}{U_\infty} + a\right)\left[a\left(\frac{U}{U_\infty} + a\right) - \frac{1}{r/R}\frac{dU/U_\infty}{dr/R}\int_0^{r/R} \frac{r_1}{R}\, a\left(\frac{r_1}{R}\right) d\frac{r_1}{R}\right] \frac{r}{R}\, d\frac{r}{R} \tag{1.31}$$

and the apparent efficiency becomes

$$\eta_a = \frac{1}{2\pi}\frac{(K_T/J_A^2)}{(K_Q/J_A^3)} \quad . \tag{1.32}$$

Finally it should be noted that the design problem of finding u_∞ , when the load distribution $\Delta p(r)$ is given, can be solved by casting Eq.(1.12) into the form

$$\frac{\Delta p}{\rho} = U u_\infty + \frac{u_\infty^2}{2} - \frac{1}{r}\frac{dU}{dr}\int_0^r r_1\, u_\infty dr_1 \tag{1.33}$$

which follows from the definition (1.6). This integral equation for u_∞ can be solved by iteration in the following way: Ignore the shear term and solve the resulting quadratic for u_∞ at each r . The integral can then be evaluated and the quadratic solved again including it. This

procedure is repeated to convergence. Equation (1.33) can also be expressed in non-dimensionless form:

$$\frac{dC_T}{dr/R} = 8\left\{ a \frac{r}{R} \frac{U}{U_\infty} + \frac{r}{R} a^2 - \frac{dU/U_\infty}{dr/R} \int_0^{r/R} \frac{r_1}{R} a \frac{dr_1}{R} \right\} \tag{1.34}$$

where

$$\frac{dC_T}{dr/R} = 2\left(\frac{r}{R}\right) \frac{\Delta p}{\frac{1}{2} \rho U_\infty^2} \tag{1.35}$$

is the thrust loading coefficient per unit radius.

II. THE GENERAL MOMENTUM THEORY

In the axial momentum theory as presented in Section I, all rotational motions have been ignored. In the general momentum theory we consider them, and hence we consider the radial momentum equation, viz.,

$$\left[v \frac{\partial v}{\partial r} - \frac{w^2}{r} + u \frac{\partial v}{\partial x}\right] = - \frac{1}{\rho} \frac{\partial p}{\partial r} \quad . \tag{2.1}$$

Here w is the azimuthal component of velocity. Now it is known from the constancy of angular momentum that ahead of the propeller there is no rotation, while behind it the propeller imparts a rotational velocity w to the fluid. Since v and u are continuous across the propeller disc, we obtain for the jump in pressure

$$\frac{1}{\rho} \frac{d\Delta p}{dr} = \frac{w^2}{r} \quad . \tag{2.2}$$

In a coordinate system rotating with the propeller blades, the flow will be steady. Hence, let

$$\begin{aligned} w &= -\Omega r + w' \\ p &= \frac{\Omega^2 r^2 \rho}{2} - p' \end{aligned} \tag{2.3}$$

Equation (2.2) then becomes

$$\frac{1}{\rho} \frac{d\Delta p}{dr} = 2\Omega w' - \frac{w'^2}{r} \quad . \tag{2.4}$$

Now let $w' = \omega r$, and integrate with respect to r. Then, if ω is a slowly varying function of r*, it is easy to show that approximately

$$\Delta p = \rho \omega r^2 (\Omega - \frac{\omega}{2}) \tag{2.5}$$

where the primes have been dropped as superfluous. This relationship is the same as that developed for a propeller in a uniform stream. Following Glauert,[6] we now let

$$\omega = 2 a' \Omega \tag{2.6}$$

---

*This approximation is also inherent in the momentum theory in a uniform flow as pointed out by von Mises,[8] although Glauert is tacit on the matter.

where a' is the rotational interference factor. Then,

$$\Delta p = 2\rho\Omega^2 r^2 a'(1-a') \tag{2.7}$$

and this expression for the jump in pressure across the actuator disc must be the same as that developed from the axial momentum theory as given in Eq.(1.12). Thus, upon substituting Eq.(1.27), we find the following relationship between the rotational and axial interference factors:

$$U_\infty^2 \left[\frac{U}{U_\infty} a + a^2 - \frac{1}{r}\frac{d(U/U_\infty)}{dr}\int_0^r a\, r_1\, dr_1\right] = \Omega^2 r^2 a'(1-a') \quad . \tag{2.8}$$

This relationship differs from the corresponding relation for uniform flow in two respects: The terms U/U_∞ in the first term on the left-hand side accounts for the parametric variation in the velocity across the wake, while the term proportional to $\frac{d(U/U_\infty)}{dr}$ accounts for the shear.

The torque may be obtained by considering the mass flow multiplied by the moment of the rotational velocity, thus

$$Q_a = \rho\int (U+u_0)\, w\, r\, dS \quad . \tag{2.9}$$

Upon substituting relationships already established or defined, there is obtained

$$Q_a = 4\pi\rho\,\Omega\int_0^R \left(U + \frac{u_\infty}{2}\right) a' r^3 dr \quad . \tag{2.10}$$

The apparent efficiency is the ratio of apparent work to the power required to generate it, i.e.,

$$\eta_a = \frac{U_\infty T_a}{\Omega\, Q_a} \quad . \tag{2.11}$$

We will now determine the condition to be satisfied for the propeller of maximum efficiency, i.e., we will minimize the power for a given thrust. Thus we set the first variation of $\Omega Q_a + \lambda U_\infty T_a$ equal to zero where, again, λ is a Lagrange multiplier. For this purpose we use the form (1.20) for the apparent thrust. Then

$$\Omega\delta Q_a + \lambda U_\infty \delta T_a = 4\pi\rho\Omega^2\left\{\int_0^R \left(U + \frac{u_\infty}{2}\right) r^3 \delta a' dr + \frac{1}{2}\int_0^R r^3 a' \delta u_\infty dr\right\}$$
$$+ 2\pi\lambda U_\infty \rho \int_0^R (V + u_\infty)\,\delta u_\infty r\, dr = 0 \quad . \tag{2.12}$$

We set $u_\infty = 2U_\infty a$, as before; then by virtue of Eq.(2.8), we can establish the following relationship between $\delta a'$ and δa

$$U_\infty^2\left[\left(\frac{U}{U_\infty} + 2a\right)\delta a - \frac{1}{r}\frac{d(U/U_\infty)}{dr}\int_0^r \delta a r_1 dr_1\right] = \Omega^2 r^2(1-2a')\delta a' \quad (2.13)$$

Upon substituting into Eq.(2.12), we obtain

$$\int_0^R \frac{(U+U_\infty a)}{1-2a'} r\, U_\infty^2 dr \left\{\left(\frac{U}{U_\infty} + 2a\right)\delta a - \frac{1}{r}\frac{d}{dr}\left(\frac{U}{U_\infty}\right)\int_0^r \delta a r_1 dr_1\right\}$$

$$+ \Omega^2 U_\infty \int_0^R r^3 a' \delta a dr + \lambda U_\infty^2 \int_0^R (V + 2U_\infty a)\delta a r dr = 0 \quad . \quad (2.14)$$

Upon interchanging the order of integration in the second term, we find

$$\int_0^R \left\{\frac{(U+U_\infty a)}{1-2a'} U_\infty^2\left(\frac{U}{U_\infty} + 2a\right) - \int_r^R \frac{(U+U_\infty a)}{1-2a'} U_\infty^2 \frac{d}{dr_1}\left(\frac{U}{U_\infty}\right) dr_1\right.$$

$$\left. + \Omega^2 U_\infty r^2 a' + \lambda U_\infty^2 (V + 2U_\infty a)\right\}\delta a r dr = 0 \quad . \quad (2.15)$$

Then, upon invoking the fundamental lemma of the calculus of variations, it is found that the condition for maximum apparent efficiency is

$$\frac{\left(\frac{U}{U_\infty} + a\right)\left(\frac{U}{U_\infty} + 2a\right)}{1-2a'} - \int_{r/R}^1 \frac{\left(\frac{U}{U_\infty} + a\right)\frac{d}{dr_1}\left(\frac{U}{U_\infty}\right)}{1-2a'} dr_1$$

$$+ \frac{\Omega^2 r^2}{U_\infty^2} a' + \lambda\left[2\frac{U}{U_\infty} - \frac{U_R}{U_\infty} + 2a\right] = 0 \quad . \quad (2.16)$$

This equation is a generalization to shear flow of the condition for maximum efficiency obtained by Betz over fifty years ago. It can be seen that it reduces to Betz' condition in the case of uniform flow. For by setting $U = U_R = U_\infty = \text{constant}$, it becomes

$$\frac{(1+a)(1+2a)}{1-2a'} + \frac{\Omega^2 r^2}{U_\infty^2} a' + \lambda(1+2a) = 0$$

which, upon dividing by (1+2a), is identical to Betz' condition as quoted by

Glauert[6] if we set $\lambda = -C$. As in the case of axial momentum, the presence of the integral terms means that the optimum distribution of the interference factors depends not only on local conditions but also on conditions elsewhere on the actuator disc, and this dependence is caused by the effect of shear.

In order to determine the axial and radial interference factors for maximum apparent efficiency, it is necessary to solve Eqs.(2.16) and (2.8) simultaneously. These constitute a pair of simultaneous nonlinear integral equations. The pair can be converted into a two-point boundary value problem by using the definition (1.6) in (2.8) and differentiating (2.16) with respect to r . One condition to be satisfied may be obtained by setting $r=R$ in Eq.(2.16) and another by setting $r=0$ in Eq.(2.8). Techniques for solving two-point boundary value problems are readily available (see, e.g., Roberts and Shipman[9]) and they can be applied to yield the solution. It would appear to be simpler, however, to obtain solutions directly from the integral equations themselves. Since the shearing term is likely to be small, the following scheme has been employed:

First, assume a value for λ (it can be seen from the limit of uniform velocity that λ is negative). Next, solve the pair of equations as simultaneous algebraic equations at each value of r/R ignoring the shear terms. Use the values of a and a' thus obtained to determine the shear terms and solve the equations again including the shear terms. Repeat this procedure until the results converge. A computer program has been prepared based on this iterative procedure.

The thrust coefficient is to be calculated using Eq.(1.30), while the torque coefficient is found from Eq.(2.10) which, in coefficient form, becomes

$$K_Q = \frac{\pi^3 J_A}{2} \int_0^1 \left(\frac{U}{U_\infty} + a\right) a' \left(\frac{r}{R}\right)^3 d\,\frac{r}{R} \quad . \tag{2.17}$$

The apparent efficiency is then given by Eq.(1.32), repeated here in slightly different form:

$$\eta_a = \frac{J_A K_T}{2\pi K_Q} \quad . \tag{2.18}$$

III. RESULTS OF A SAMPLE COMPUTATION

A computer program for determining the optimum distribution of the axial and rotational interference factors has been prepared and applied to a particular measured wake determined from model tests.* The ratio of stream velocity to forward speed in the wake is presented in Table 1 and a listing of the program in Fortran IV is given in the Appendix.

TABLE 1

MODEL WAKE VELOCITY DISTRIBUTION

r/R	U/U_∞
0.	.225
.05	.242
.10	.261
.15	.273
.20	.307
.25	.333
.30	.359
.35	.382
.40	.413
.45	.440
.50	.473
.55	.508
60	.541
.65	.577
.70	.610
.75	.640
.80	.671
.85	.696
.90	.720
.95	.743
1.00	.768

*supplied by the sponsoring agency.

Self-powered model tests had been carried out and it was found that this wake was appropriate for the advance ratio J_A=.891 and that the thrust coefficient $K_T = 0.17$. In solving the pair of equations (2.8) and (2.16), it is not possible to specify K_T, but instead the Lagrange multiplier λ must be specified; consequently, a value of λ was assumed and the resulting value of K_T computed. Then λ was adjusted using a manual search procedure until K_T became equal to 0.17. It is, of course, possible to automate this search procedure but for the limited number of computations carried out here, this would have been unnecessarily elaborate.

The pair of equations (2.8) and (2.16) were solved a second time with the shear terms omitted and $2U-U_R$ replaced by U. This corresponds to considering U as a function of r only parametrically, and thus in this case all effects of shear are suppressed. A comparison of the results both including and excluding the effects of shear are shown in Figures 1, 2, and 3. The distribution of thrust loading was calculated using Eq.(1.34).

It can be seen from Figure 1 that the distribution of the rotational interference factor is strongly dependent on shear only for the inboard sections which do not carry a significant portion of the load. On the other hand, from Figure 2 the optimum distribution of the axial interference factor is strongly dependent on shear for the outboard sections and suggests that a wake-fitted propeller designed without accounting for shear is pitched at the wrong angle to obtain maximum efficiency. Indeed, the optimum load distribution as presented in Figure 3 shows that the tips should be more heavily loaded when shear is accounted for than when it is not.

The measured model wake given in Table 1 was processed* to determine the wake on the full-scale vehicle and the full-scale wake data are presented in Table 2. It was assumed that the advance ratio and thrust coefficient are the same at full-scale as for the model tests and plots of the rotational interference factor, the axial interference factor and the optimum load distribution for these conditions are presented in Figures 4, 5 and 6, respectively, both with and without shear. The relative effects of shear are seen to be similar to the results for the model wake.

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*by the sponsoring agency.

TABLE 2

FULL-SCALE WAKE VELOCITY DISTRIBUTION

r/R	U/U_∞
0.	.270
.05	.291
.10	.314
.15	.328
.20	.369
.25	.394
.30	.420
.35	.442
.40	.473
.45	.500
.50	.533
.55	.567
.60	.600
.65	.636
.70	.669
.75	.700
.80	.731
.85	.756
.90	.779
.95	.799
1.00	.820

It is also interesting to compare the optimum distributions for the model wake with the optimum distributions at full-scale including shear in both cases. These comparisons are shown in Figures 7, 8, 9. From Figure 7 we see that the rotational interference factors are virtually the same between model and full-scale at all radii. From Figure 8 we see that the axial interference factor has a small but significant difference at all radii and this suggests that even when shear is accounted for, the design of a full-scale wake-adapted propeller must be pitched differently from its

model prototype in order to optimize its performance. This is borne out in Figure 9 where a comparison of the optimum load distribution for model and full-scale indicates that the tips should be more heavily loaded at the expense of inboard stations in full-scale in comparison with model scale.

Since the momentum theory does not allow the load to drop to zero at the tips as it must, these conclusions may require some modification. Nevertheless, momentum theory in combination with blade element theory has served as a fairly reliable design tool in the past and the conclusions drawn here are very likely to be qualitatively correct. It is planned, as mentioned in the Introduction, to examine a more sophisticated theory for a special family of wake profiles in which the tip effect including shear is accounted for.

Finally it should be remarked that the optimum apparent efficiency at full-scale is smaller than at model scale and that the effect of including shear is to reduce the optimum efficiency from that which would be obtained if shear were neglected.

IV. ACKNOWLEDGMENTS

The author would like to thank Professor John P. Breslin who suggested the problem and with whom he had several fruitful discussions. Thanks are also extended to Mr. Ping Liao who prepared the computer program and to Mrs. Katherine Wilkens who typed the manuscript.

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APPENDIX

Given below is a list of computer symbols and corresponding mathematical symbols for the program used to solve Eqs.(2.8) and (2.16) simultaneously:

COMPUTER SYMBOL	MATHEMATICAL SYMBOL
AJ	initial value of J_A
PNI	initial value of λ
NAAJ	number of values of J_A
NNP	number of values of λ
DAJ	increment of J_A
DPN	increment of λ
UR(I)	values of U/U_∞ given in Table 1 or Table 2
URT	value of U/U_∞ at $r/R = 1$
H	increment of r/R
URP(I)	$d(U/U_\infty)/d(r/R)$
X1	a
X2	a'

NOTES ON THE PROGRAM

QTFE	IBM 360-SSP integration subroutine using trapezoid rule
DET3	IBM 360-SSP numerical differentiation subroutine using central differences
SUBROUTINE PG1	subroutine to solve Eqs.(2.8) and (2.16) simultaneously at given value of r/R for given values of BOT and TOP
BOT	integral term in (2.8)
TOP	integral term in (2.16)

LISTING OF COMPUTER PROGRAM

```
.TY PG2F
      DIMENSION UR(21),URP(21),X1(21),X2(21),TOP(21),BOT(21)
      DIMENSION FT(21),FB(21)
      DIMENSION  FKJ(21),FKQ(21)
     DIMENSION CTF(21)
      DOUBLE PRECISION SHIP
      COMMON URT,PI,PN,PJ42
      DATA X1,X2/42*0.0/
      DATA PI/3.1415926/
      CALL IFILE(21,'GOOD1')
      ACCEPT 105,SHIP
105   FORMAT(A8)
      ACCEPT 100,AJ
      ACCEPT 100,PN1
      ACCEPT 101,NAAJ,NNP
      ACCEPT 100,DAJ
      ACCEPT 100,DPN
101   FORMAT(2I)
      READ(21,100) (UR(I),I=1,21)
      URT=UR(21)
100   FORMAT(F)
      H=.05
      CALL DET3(H,UR,URP,21,IER)
      IF(IER.NE.0) CALL EXIT
      TYPE 950,SHIP
950   FORMAT('1',5X,'OPTIMUM LOADING WITH SHEAR - 'A8//1X,
     1 '        JA'8X'LA'8X'KT'8X'KQ'8X'CT'7X'ETA')
      DO 991 NAJ=1,NAAJ
      PJ4=PI/AJ
      PJ42=PJ4*PJ4
      PN=PN1
      DO 990 NP=1,NNP
      X1(21)=.5
      X2(21)=0.
      DO 90 LOOP=1,10
      DO 30 K=1,21
      FT(K)=(UR(K)+X1(K))*URP(K)/(1.-2.*X2(K))
      XK=K
            FB(K)=X1(K)*(XK-1.)*.05
30    CONTINUE
```

```
      CALL QTFE(H,FT,TOP,21)
      CALL QTFE(H,FB,BOT,21)
      DO 40 I=1,21
40    TOP(I)=TOP(21)-TOP(I)
      Y1=X1(21)
      Y2=X2(21)
      DO 50 J=1,21
      I=22-J
      XI=I
              RI=H*(XI-1.)
      CALL PG1(Y1,Y2,UR(I),URP(I),TOP(I),BOT(I),RI)
      X1(I)=Y1
      X2(I)=Y2
50    CONTINUE
90    CONTINUE
      DO 80 I=1,21
      XI=I
      R=(XI-1.)*H
      FKJ(I)=((2.*UR(I)-URT)+X1(I))*X1(I)*R
      FKQ(I)=(UR(I)+X1(I))*X2(I)*R*R*R
      IF (R.EQ.0.) GOTO 81
      CTF(I)=8.*R*(UR(I)*X1(I)+X1(I)*X1(I))-8.*URP(I)*BOT(I)
      GOTO 80
81    CTF(I)=0.
80    CONTINUE
200   FORMAT(2X,F4.2,4E16.4)
      CALL QSF(H,FKJ,TOP,21)
      CALL QSF(H,FKQ,BOT,21)
      CT=8.*TOP(21)
      FJ=PI*AJ*AJ*TOP(21)
      FQ=.5*PI*PI*AJ*BOT(21)
      EH=.5*AJ*FJ/FQ/PI
      IF(NP.GT.1) GOTO 95
      TYPE 900,AJ,PN,FJ,FQ,CT,EH
      GOTO 990
95    TYPE 905,PN,FJ,FQ,CT,EH
900   FORMAT(2X,6F10.4)
905   FORMAT(12X,5F10.4)
990   PN=PN-DPN
991   AJ=AJ+DAJ
      END
```

```
      SUBROUTINE PG1(X1,X2,UR,URP,TOP,BOT,RI)
      COMMON URT,PI,PN,PJ42
      DO 10 I=1,50
      F1=(UR+X1)*(UR+2.*X1)-TOP*(1.-2.*X2)
      F1=F1+PJ42*RI*RI*X2*(1.-2.*X2)
      F1=F1+PN*(1.-2.*X2)*(2.*UR-URT+2.*X1)
      IF(RI.EQ.0.) GOTO 30
              F2=UR*X1+X1*X1-URP*BOT/RI
      GOTO 40
30    F2=UR*X1+X1*X1
40    CONTINUE
      F2=F2-PJ42*RI*RI*X2*(1.-X2)
      FSQ=F1*F1+F2*F2
      IF(FSQ.LT.1E-12) RETURN
      F1X1=3.*UR+4.*X1+2.*PN*(1.-2.*X2)
      F1X2=2.*TOP+PJ42*RI*RI*(1.-4.*X2)
      F1X2=F1X2-2.*PN*(2.*UR-URT+2.*X1)
      F2X1=UR+2.*X1
      F2X2=-PJ42*RI*RI*(1.-2.*X2)
              XJ=F1X1*F2X2-F1X2*F2X1
      X1=X1+(F2*F1X2-F1*F2X2)/XJ
      X2=X2+(F1*F2X1-F2*F1X1)/XJ
10    CONTINUE
      RETURN
      END
```

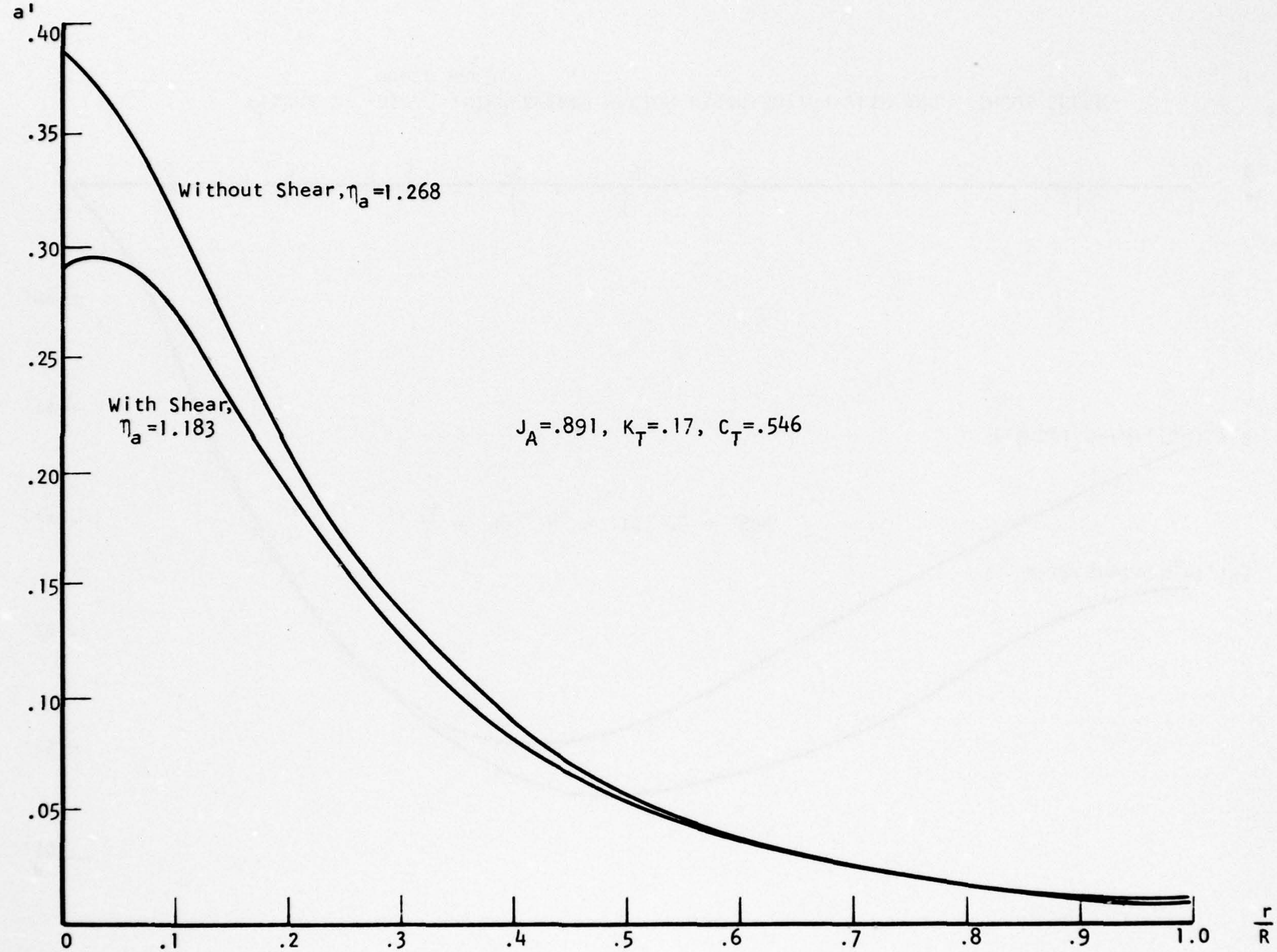

FIGURE 1. ROTATIONAL INTERFERENCE FACTOR DISTRIBUTION WITH AND WITHOUT SHEAR - MODEL WAKE

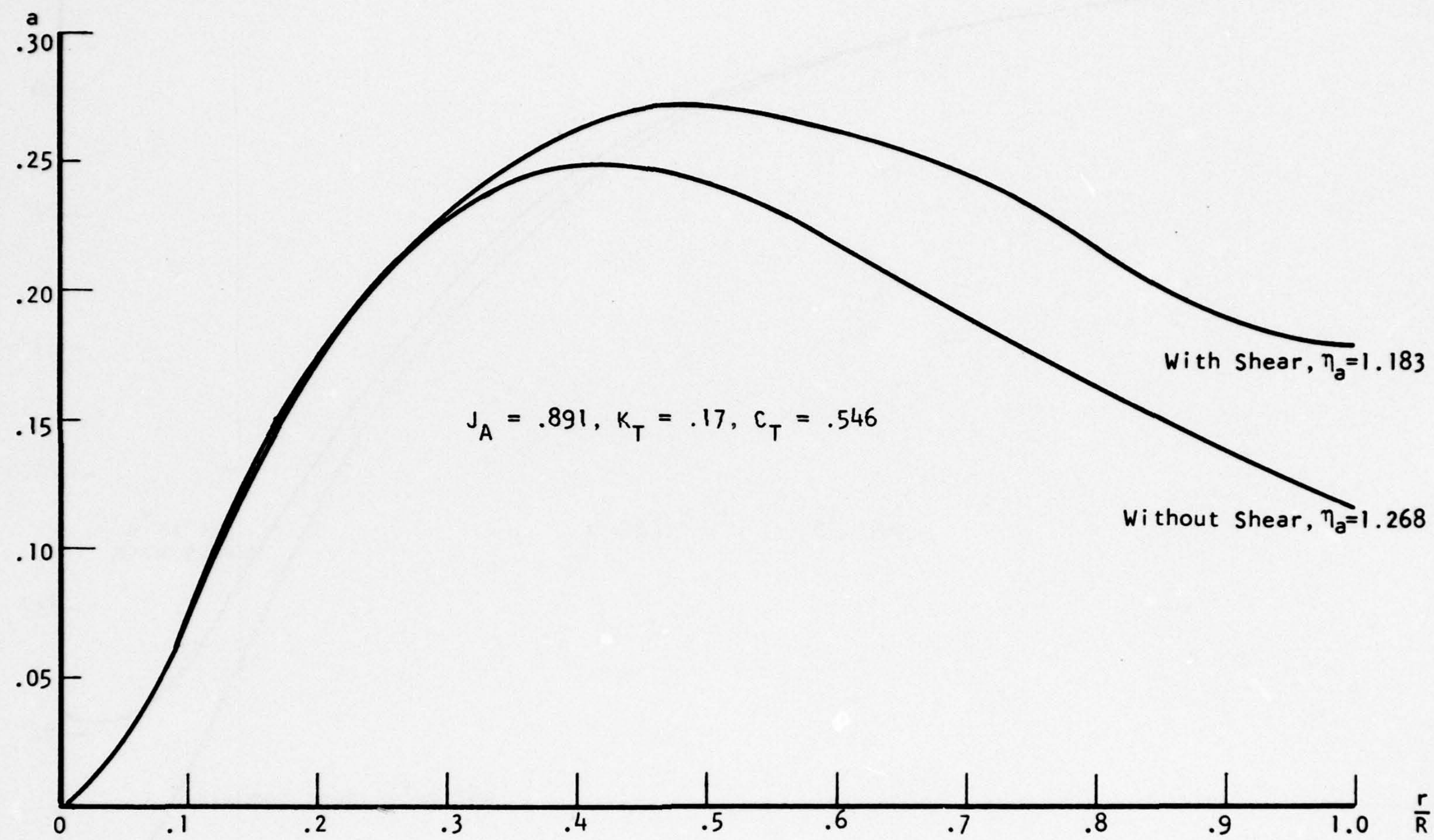

FIGURE 2. AXIAL INTERFERENCE FACTOR DISTRIBUTION WITH AND WITHOUT SHEAR - MODEL WAKE

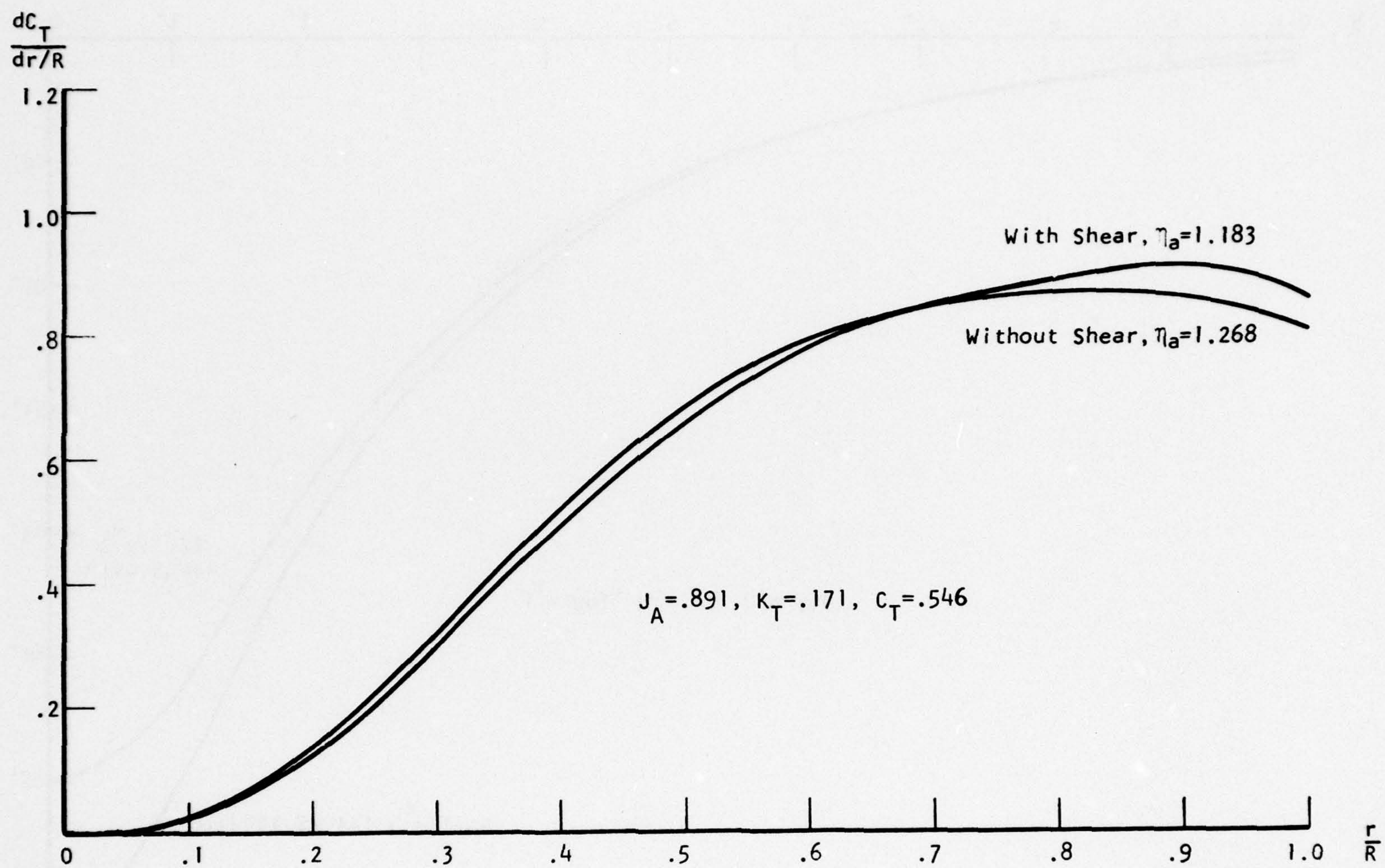

FIGURE 3. THRUST LOADING DISTRIBUTION WITH AND WITHOUT SHEAR - MODEL WAKE

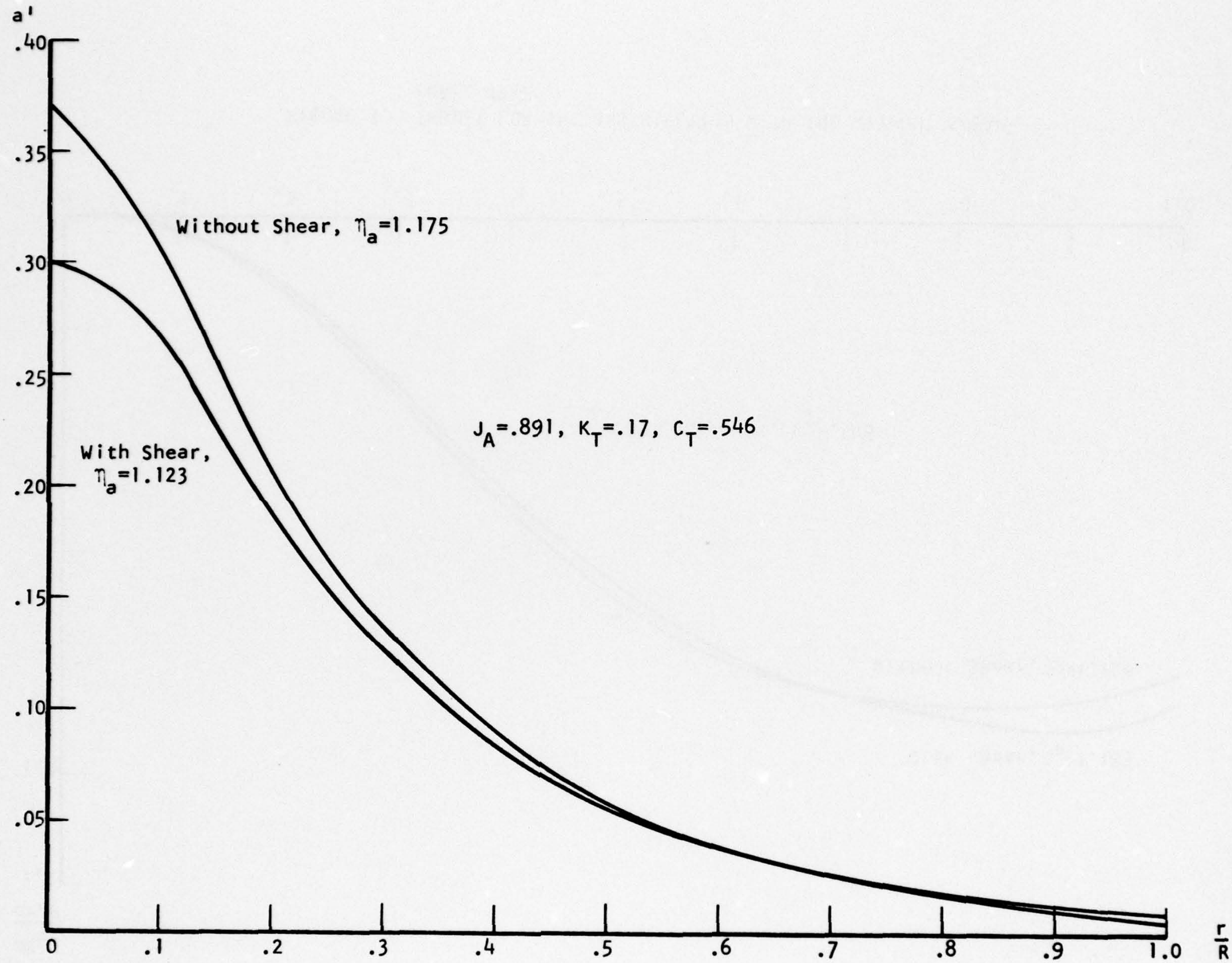

FIGURE 4. ROTATIONAL INTERFERENCE FACTOR DISTRIBUTION WITH AND WITHOUT SHEAR - FULL-SCALE WAKE

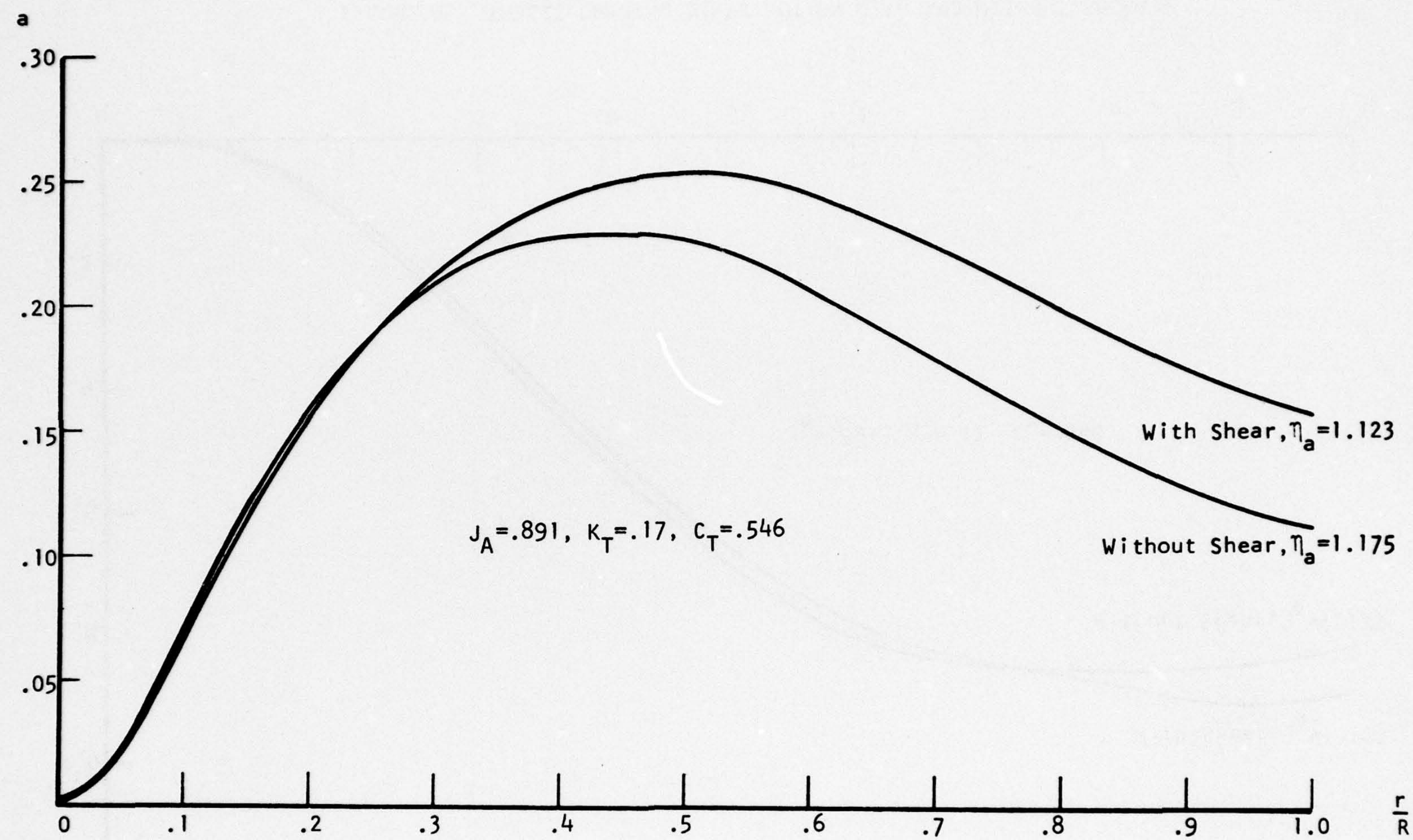

FIGURE 5. AXIAL INTERFERENCE FACTOR DISTRIBUTION WITH AND WITHOUT SHEAR - FULL-SCALE WAKE

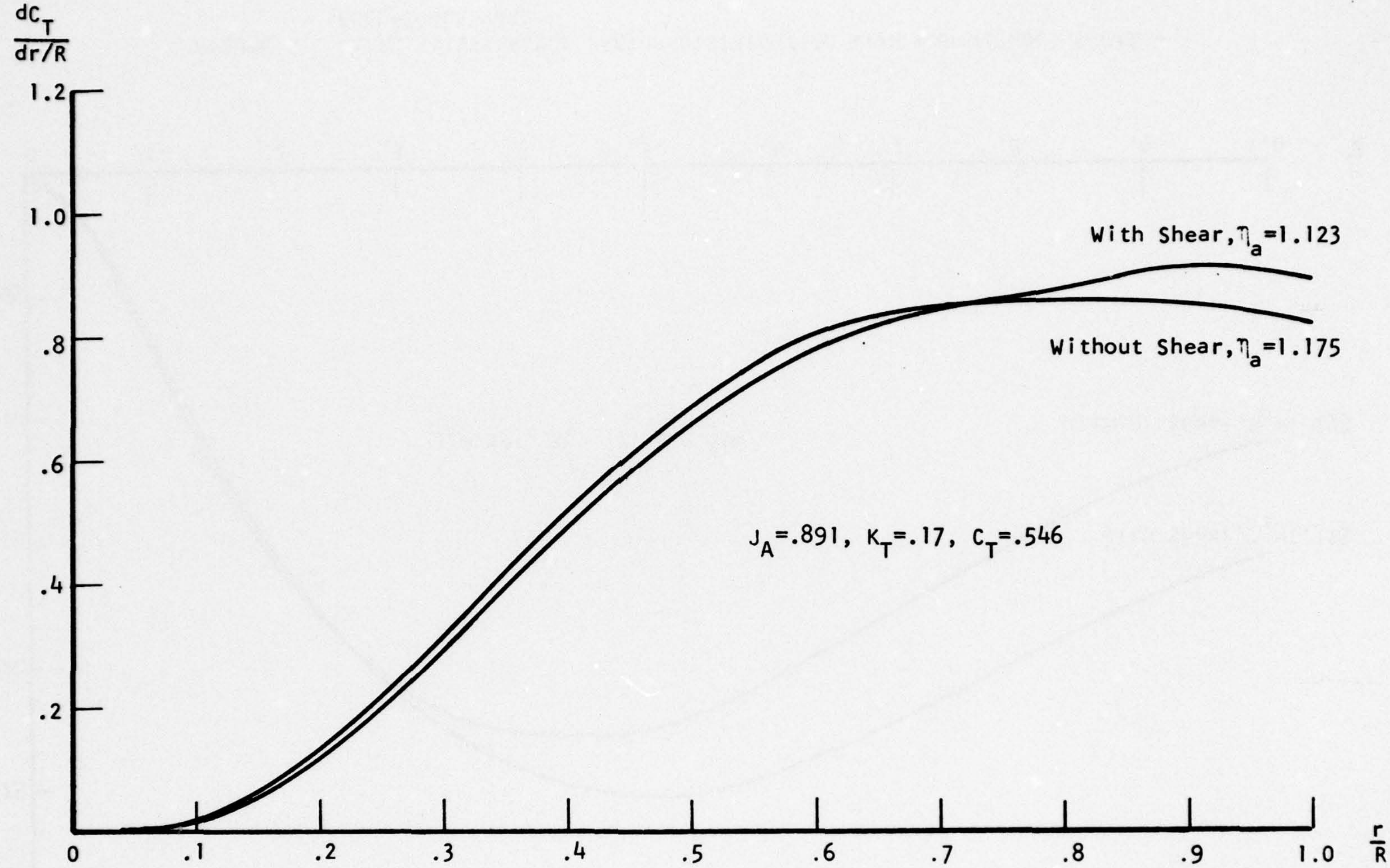

FIGURE 6. THRUST LOADING DISTRIBUTION WITH AND WITHOUT SHEAR - FULL-SCALE WAKE

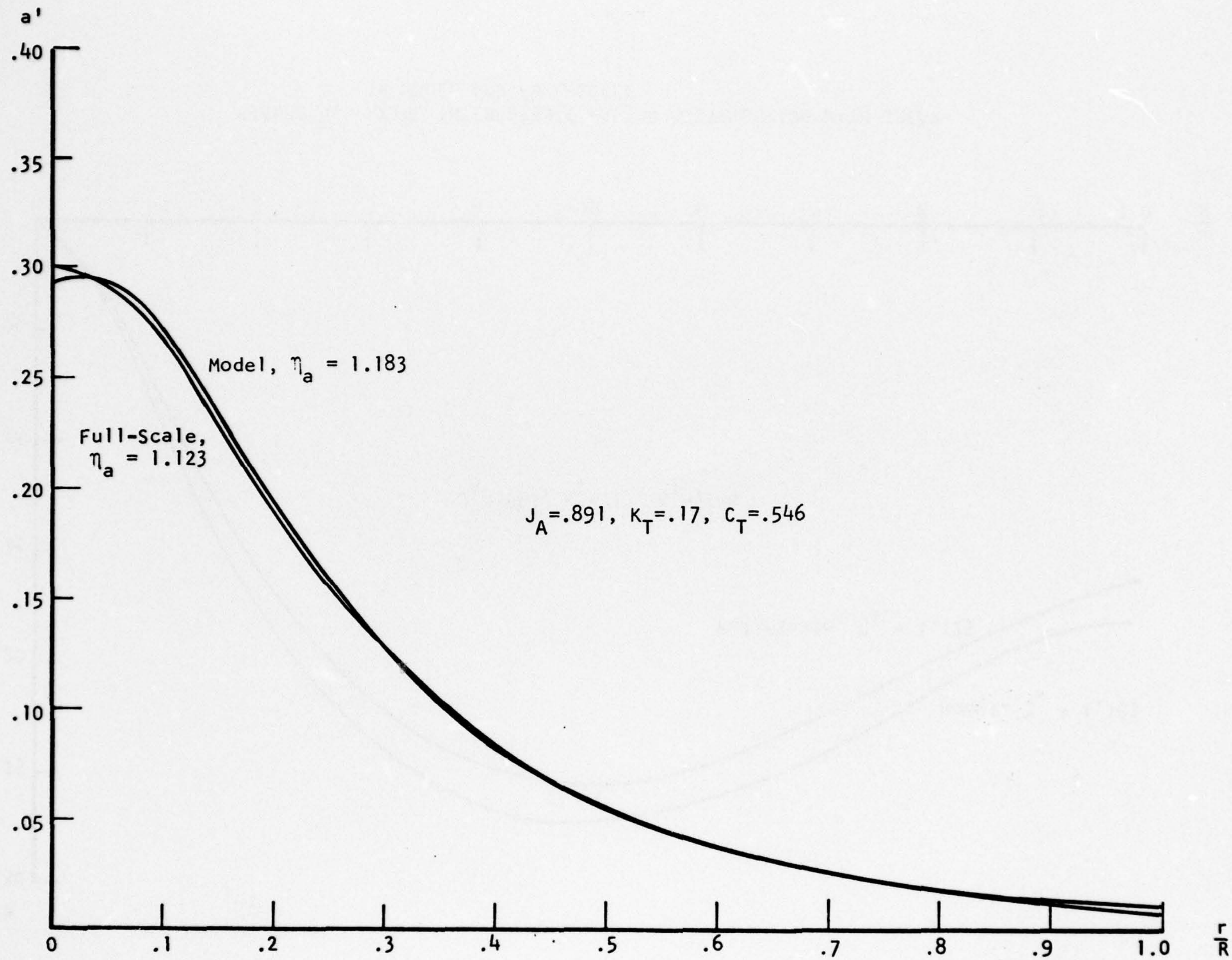

FIGURE 7. ROTATIONAL INTERFERENCE FACTOR DISTRIBUTION WITH SHEAR IN MODEL AND FULL-SCALE

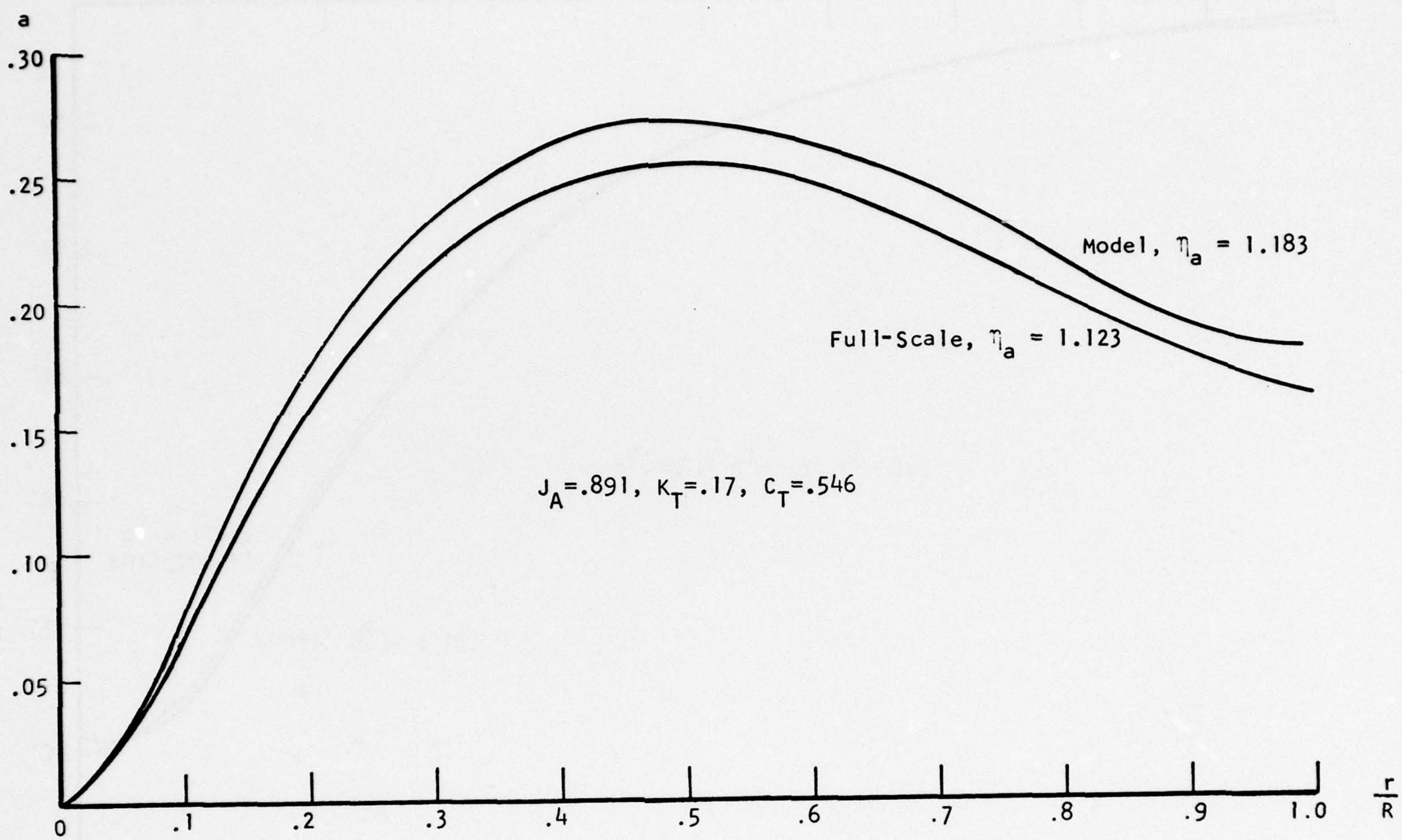

FIGURE 8. AXIAL INTERFERENCE FACTOR DISTRIBUTION WITH SHEAR IN MODEL AND FULL-SCALE

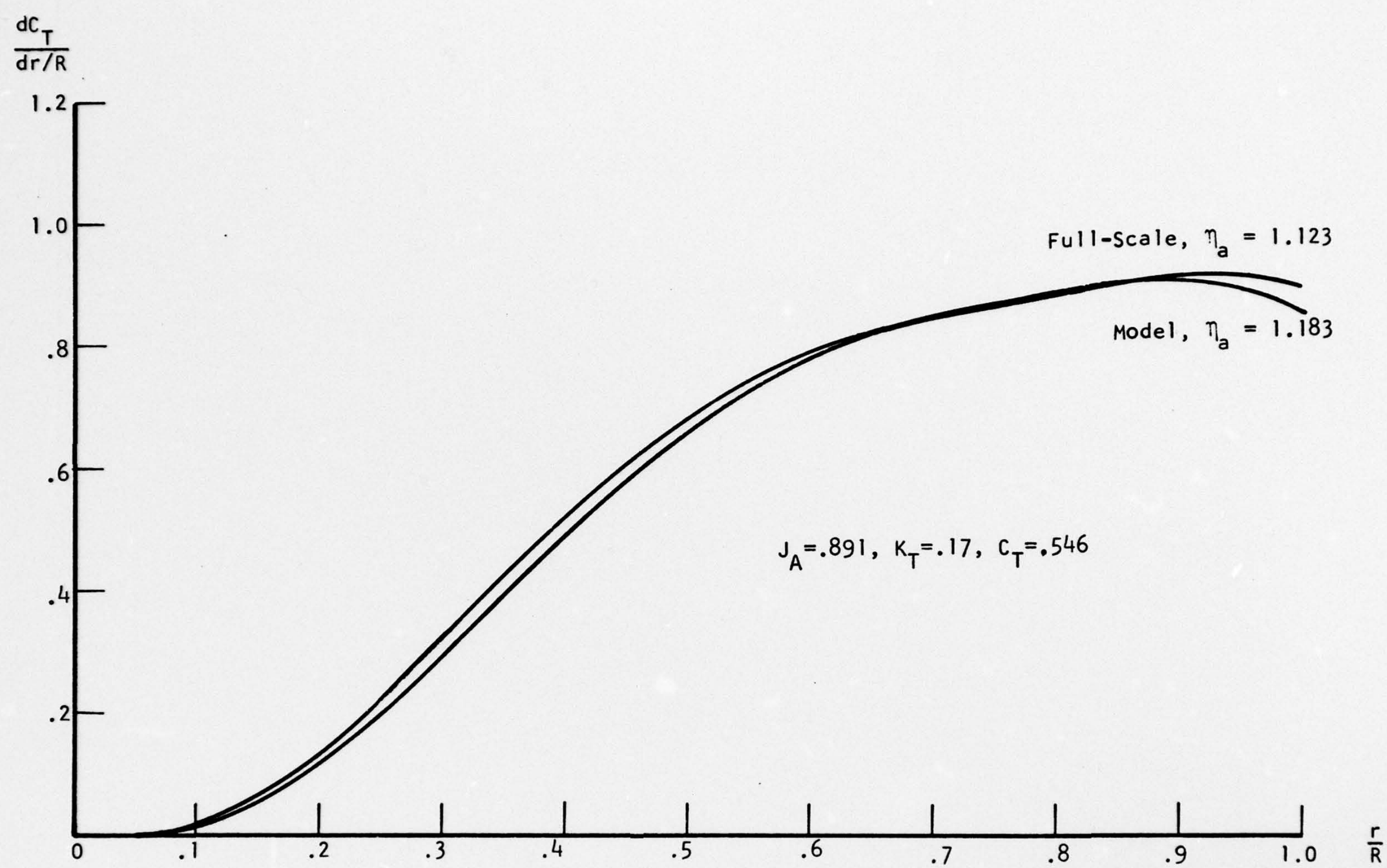

FIGURE 9. THRUST LOADING DISTRIBUTION WITH SHEAR IN MODEL AND FULL-SCALE

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